Integral calculus

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Definition 1.
a. $\int_a^a f(x)dx = 0$
b. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

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Theorem 1.
a. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$. And if $k=-1$, then $\int_a^b -f(x)dx = -\int_a^b f(x)dx$
b. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
c. If $f(x) \geq g(x)$ on $[a,b]$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$ Let $g(x)=0$. Then, $f(x) \geq 0$ on $[a,b]$ implies $\int_a^b f(x)dx \geq 0$
d. If $\max f$ and $\min f$ are the maximum and minimum values of f on $[a,b]$, then

$$\min f \cdot (b-a) \leq \int_a^b f(x)dx \leq \max f \cdot (b-a)$$

e. If f if integrable on the intervals between a, b and c, then

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

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Theorem 2 is called *2*.

Theorem 2. If f is continuous on the closed innterval $[a,b]$, then at some point c in the interval $[a,b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

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Theorem 3. The *average-* or *mean value* of an integrable function f on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)dx$$

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Theorem 4. If f has a constant value c on $[a,b]$, then

$$\int_a^b f(x)dx = \int_a^b c\,dx = c(b-a)$$

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Theorem 5. If f is continuous on $[a,b]$, then the function $F(x) = \int_a^x f(t)dt$ has a derivative at every point on $[a,b]$ and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

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Theorem 6. If f is continuous at every point of $[a,b]$ and F is an antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

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Bibliography

George B Thomas, Jr and Ross L Finney. *Calculus and analytic geometry.* 8^{th}, 1992